

It is assumed that the nominal voltage V , power P , slip s , efficiency η , power factor $\cos\phi$ and the locked rotor current I_s are known. The power is the “useful” electrical power, that is, the power consumed in the load resistance $R_2(1-s)/s$ of the equivalent circuit. The efficiency η is the electrical efficiency, i.e. the “useful” electrical power divided by the total power taken by the motor.

The well known equivalent circuit of the motor is given in figure below

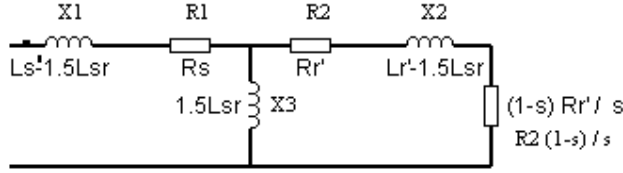


Figure: The equivalent circuit of the induction motor

Note, one has to assume something about X_1 , because there are too many unknowns compared with the number of equations. The unknowns are X_1 , X_2 , X_3 , R_1 , R_2 . Equations can be written for power, efficiency, power factor, and the locked rotor current.

The resistance R_1 is solved first. When the useful power per phase is $P/3$, the losses in R_2 are $(P/3) \cdot s / (1-s)$. The losses in R_1 are equal to $I^2 R_1$. The total power taken by the motor per phase is thus

$$U I \cos\phi = I^2 R_1 + (P/3) / (1-s)$$

where U is the phase to neutral voltage and I is the absolute value of the nominal current. Because $P/3$ is equal to the power taken by the motor times efficiency, $P/3 = \eta U I \cos\phi$, the resistance R_1 is equal to

$$R_1 = (U \cos\phi / I) (1 - \eta / (1-s)) \quad (1)$$

The solution of the other impedances is a bit more complicated. Let us first define the “apparent impedance” $R_a + j X_a$ as $i = U / (R_a + j X_a)$ where i is the nominal complex current of the motor. Because the voltage, power, etc are known for the motor, the numerical values of the current i and the apparent impedance can be calculated. Equations for the resistance R_a and reactance X_a of the apparent impedance can be derived using the equivalent circuit of the motor, as

$$R_a = R_1 + R \frac{X_3^2}{[R^2 + (X_2 + X_3)^2]} \quad \text{where } R = R_2/s \quad (2)$$

$$X_a = X_1 + [R^2 X_3 + X_2 X_3 (X_2 + X_3)] / [R^2 + (X_2 + X_3)^2] \quad (3)$$

Next, R^2 is solved from (2) and substituted into (3). After some simplifications and assuming $X_1 = X_2$, an equation is obtained for R , as

$$R (X_3 + X_2 - X_a) = (X_2 + X_3) (R_a - R_1) \quad , \text{ where } R = R_2/s \quad (4)$$

R is solved from (4), and substituted into (2), which gives an equation for X_3 , as

$$X_3^2 = (R_a - R_1)^2 (X_2 + X_3) / (X_2 + X_3 - X_a) + (X_2 + X_3) (X_2 + X_3 - X_a) \quad (5)$$

After these calculations, R_1 is known, and the resistance R_2 and the reactances X_1, X_2, X_3 can be expressed as a function of $X_2 + X_3$, which is still unknown. The sum $X_2 + X_3$ must be solved iteratively from the equation for the locked rotor current. This completes the solution of the impedances.